Seasonal Adjustment Subject to Frequency Aggregation Constraints

Osbert Pang
U.S. Census Bureau

November 20, 2019

Acknowledgements

Joint work with Tucker McElroy and Brian Monsell.

Introduction

- Presence of residual seasonality in published GDP figures
- Renewed interest in seasonality diagnostics and seasonal adjustment at the Bureau of Economic Analysis (BEA)
- Preliminary findings at BEA indicated that residual seasonality could occur as a result of aggregating monthly source data to quarterly frequency – Moulton (2016)
- Demonstration of phenomenon through simulations, theoretical models – McElroy (2016)

Indirect vs Direct Seasonal Adjustment

- Suppose we have (raw) monthly source data available, and we wish to obtain a quarterly seasonal adjustment

- **Indirect adjustment** – we seasonally adjust the monthly source data and aggregate the adjustment

- **Direct adjustment** – we aggregate the monthly source data and seasonally adjust the aggregate

- More control over outcomes with direct adjustment, so easier to ensure adequacy (i.e., the resulting adjustment does not exhibit seasonality) ...

Indirect vs Direct Seasonal Adjustment (2)

- But direct adjustment generally not equal to indirect adjustment (i.e., accounting relationships not preserved)

- If monthly seasonally adjusted numbers are published, then having quarterly numbers that do not satisfy this aggregation requirement is a drawback

- Further complication: sometimes, the monthly raw data is not available; i.e., the data at hand is a monthly seasonal adjustment, making it impossible to compute a direct adjustment

- That is, sometimes, indirect adjustment is the only option, and this adjustment may not necessarily be adequate

Seasonality in Frequency Aggregated Series

- "Frequency-aggregated seasonality" – when a change in sampling frequency via (flow) aggregation exhibits seasonality in the resulting aggregate

- E.g., we have a monthly time series that shows no seasonality; when aggregated to quarterly frequency, seasonality is observed

- Alternately, we have a monthly time series that is seasonal, is (adequately) seasonally adjusted, but seasonality is observed in the quarterly aggregate of the monthly adjustment

- Direct adjustment of the quarterly series would remove seasonality in either scenario, but then the direct adjustment will not equal the aggregate of the monthly adjustment.

Benchmarking Methodology

- Benchmarking problem: we have a time series sampled at a high and low frequency; for convenience, we can assume these are monthly and quarterly, respectively

- Literature on benchmarking is extensive, but issue of adequacy is not usually addressed

- Goal here is to try to adjust monthly and quarterly data such that both sets of seasonal adjustments are adequate

Some Notation

- Let the monthly series be denoted $\{X_{t,m}\}$ and its quarterly counterpart $\{X_{i,q}\}$, where $t = 3i + j$ for $j = 1, 2, 3$, where the data satisfy the following frequency aggregation property for quarter i:

$$X_{i,q} = X_{3i+1,m} + X_{3i+2,m} + X_{3i+3,m}$$

- Denote direct adjustments with N, so $\{N_{t,m}\}$ and $\{N_{i,q}\}$ – these may, but generally will not, satisfy the aggregation property above

- If not, we want modifications $\{Y_{t,m}\}$ and $\{Y_{i,q}\}$ that do preserve this property, are close to the direct adjustments, and are adequate

- If $\{N_{t,m}\}$ is available, but not $\{N_{i,q}\}$, then define $\{N_{i,q}\}$ as follows:
  - Aggregate $\{N_{t,m}\}$, test aggregate for seasonality
  - If adequate, done; else, seasonally adjust and use resulting adjustment as $\{N_{i,q}\}$

United States Census Bureau

Some Notation (2)

- To minimize discrepancy between $\{Y_{t,m}\}$ and $\{N_{t,m}\}$, and between $\{Y_{i,q}\}$ and $\{N_{i,q}\}$, while preserving the frequency aggregation property and ensuring adequacy of both $\{Y_{t,m}\}$ and $\{Y_{i,q}\}$, amounts to minimizing the following expression for each quarter i:

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} \left(N_{3i+j,m} - Y_{3i+j,m}\right)^2 / N_{3i+j,m}.$$

- Adequacy checked by applying some diagnostic δ to candidate solutions for monthly and quarterly series, compared against some threshold α

United States Census Bureau

What Diagnostic δ?

- McElroy (2018) used the QS diagnostic of Maravall (2012) as the diagnostic δ
- Concerns stemming from spurious detections of seasonality
- Instead, we use root diagnostic of McElroy (2019), which offers p-value for rejection of null hypothesis that seasonality is present to a given degree

Seasonality Diagnostic Based on Autoregressive Roots (McElroy, 2019)

- A causal invertible ARMA(p, q) process with MA polynomial $\theta(z)$ and AR polynomial $\phi(z)$ can also be represented using an MA(∞) $\psi(z) = \theta(z)/\phi(z)$

- A process is said to have "ρ-persistent seasonality of frequency $\omega \in [-\pi, \pi]$ (where $\rho \in (0, 1]$) iff its causal representation has coefficients $\{\psi_j\}$ with a ρ-persistent oscillatory effect of frequency $\omega \in [-\pi, \pi]$, such that $\pi(\rho^{-1} e^{i\omega}) = 0$, where $\pi(z) = 1/\psi(z)$"

- What is tested: for any given ω, the null hypothesis is

$$H_0(\rho_0) : \pi(r^{-1} e^{i\omega}) = 0 \quad \text{has solution } \; r = \rho_0$$

- The test statistic of $H_0(\rho_0)$ for a sample of size T is

$$T \left| \widehat{\pi}(\rho_0^{-1} e^{i\omega}) \right|^2$$

United States Census Bureau

… In a Simpler Setting

- Simplifying, suppose we have an AR(p) process with AR polynomial $\phi(z)$, then $\pi(z)$ in the previous expressions is replaced by $\phi(z)$

- The null hypothesis says that for a given frequency ω, there is a root for the AR polynomial $\phi(r^{-1}e^{i\omega})$ at $r = \rho_0$

- If the magnitude of the AR polynomial (or the estimated AR polynomial) evaluated at $\rho_0^{-1}e^{i\omega}$ is large, that will produce a large test statistic; i.e., it would suggest that seasonality of a degree ρ_0 is not present in the tested process

- Note that this hypothesis test is laid out opposite – the null is positing the presence of seasonality to a certain degree, instead of no seasonality

Using Root Diagnostic

- Using the diagnostic, we look at p-values as a function of seasonal persistance ρ at frequencies $2\pi/4$ (for quarterly) and $2\pi j/12$ for $j = 1, 2, \ldots, 5$ (for monthly)

- Require
$$\max_{\rho \in (0.98, 1)} p(\rho) \leq \alpha,$$
where $p(\rho)$ denotes p-value as function of ρ determined by H_0

- Above says null hypothesis of seasonality of degree ρ can be rejected at level α for all $\rho \in (0.98, 1)$ – 0.98 corresponds to substantial degree of oscillation in autocorrelation function; lowering this value requires even weaker forms of seasonality be weeded out

- Notation-wise,
$$\delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\} \leq \alpha, \quad \delta\{Y_{1,q}, \ldots, Y_{i,q}\} \leq \alpha,$$
where δ indicates the maximum of p-values (3) computed on either monthly or quarterly data

United States Census Bureau

Constrained Minimization

- Goal: minimize

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} (N_{3i+j,m} - Y_{3i+j,m})^2 / N_{3i+j,m}.$$

subject to the constraints

$$\delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\} \leq \alpha, \quad \delta\{Y_{1,q}, \ldots, Y_{i,q}\} \leq \alpha,$$

- Doable with Lagrangian techniques with inequality constraints or slack variables
- What we try: Convert constrained minimization problem into penalized minimization, iteratively increase the penalty until a solution has been achieved

Penalized Minimization

- I.e., introduce tuning parameters $\omega_m, \omega_q \geq 0$, and minimize objective function

$$\left(N_{i,q} - \sum_{j=1}^{3} Y_{3i+j,m}\right)^2 / N_{i,q} + \sum_{j=1}^{3} \left(N_{3i+j,m} - Y_{3i+j,m}\right)^2 / N_{3i+j,m}$$
$$+ \omega_m \left(\text{min}\left[\alpha - \delta\{Y_{1,m}, \ldots, Y_{3i+3,m}\}, 0\right]\right)^2$$
$$+ \omega_q \left(\text{min}\left[\alpha - \delta\{Y_{1,q}, \ldots, Y_{i,q}\}, 0\right]\right)^2$$

- Penalty terms are zero iff $\delta \leq \alpha$; solutions where $\delta > \alpha$ tend to be rejected

- Possible for adequate solutions to be obtained where $\omega_m = \omega_q = 0$, so using 0 as initial value is not unreasonable

- If candidate solution at initial values of ω_m and ω_q is not adequate at either frequency, increment both; repeat until adequate solution is achieved

General Framework

- Aggregate monthly series to quarterly series
- Use root diagnostic to determine whether either series is seasonal
- Construct indirect quarterly adjustment by aggregating monthly seasonal adjustment (or monthly raw series if deemed nonseasonal by root diagnostic)
- Use root diagnostic to determine whether monthly adjusted (or raw) series or indirect quarterly adjustment is seasonal
- If not, done; else, initialize ω_m and ω_q and begin nonlinear optimization of objective function
- Apply root diagnostic to reconciled series – if adequate, done; else, increment ω_m, ω_q and repeat

United States Census Bureau

Sample Applications

- Around 50 monthly economic series taken from some surveys conducted by U.S. Census Bureau, measuring quantities like shipments, construction spending, imports/exports

- Using $\rho \in (0.98, 1)$ calculated in 0.001 increments, majority of these series are such that null hypothesis of seasonality of degree ρ is rejected at both monthly and aggregated quarterly levels at an α of 0.1

- Some series (approx. 20–25%) where the raw monthly series is borderline seasonal (or borderline nonseasonal, and thus might be left as is), while the resulting quarterly aggregate is more noticeably seasonal

- Optimization starts with an initial value of $\omega_m = \omega_q = 0$, incrementing each by 1000 should a solution fail to satisfy the adequacy conditions

- Optimization is the major bottleneck; examples use the Bound Optimization by Quadratic Approximation (BOBYQA) algorithm of Powell (2009), as implemented in the `minqa` package in R

Example 1: Import Series

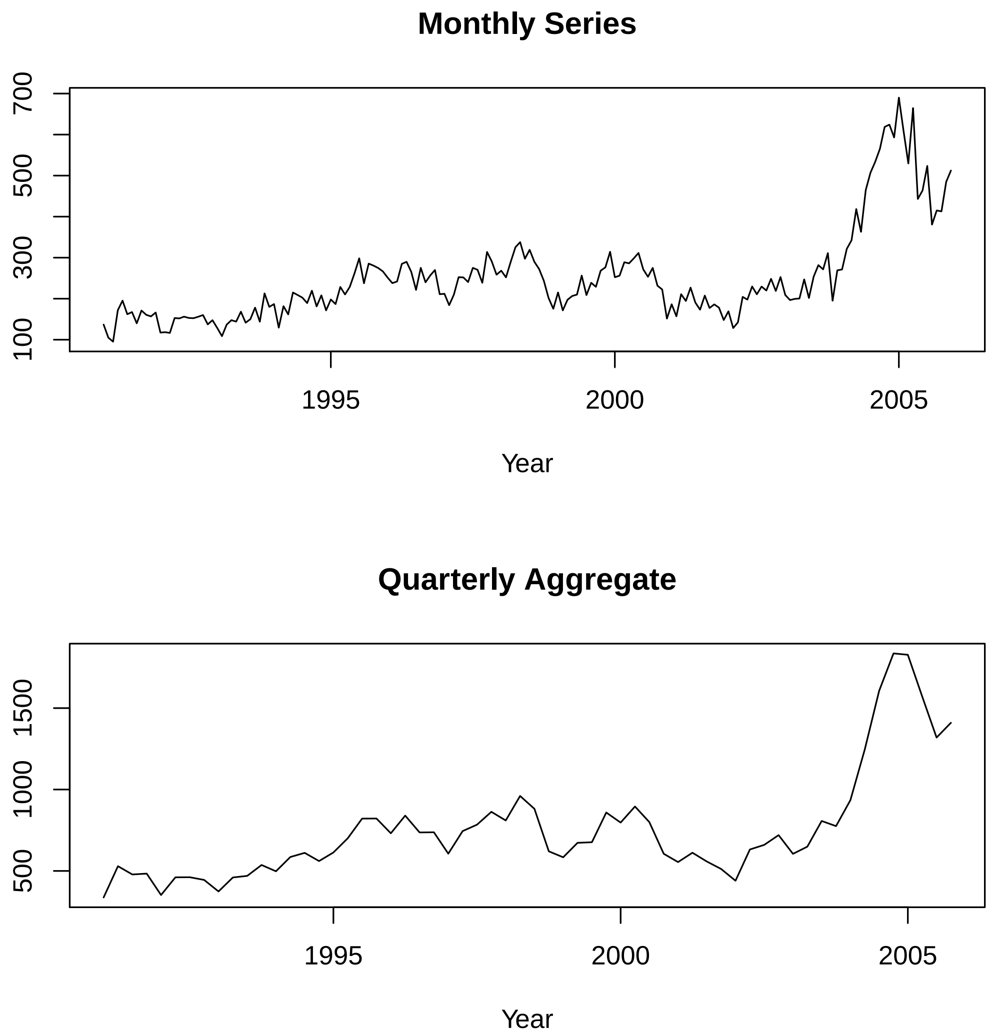


Figure 1: Monthly and quarterly aggregated series.

Example 1: Monthly

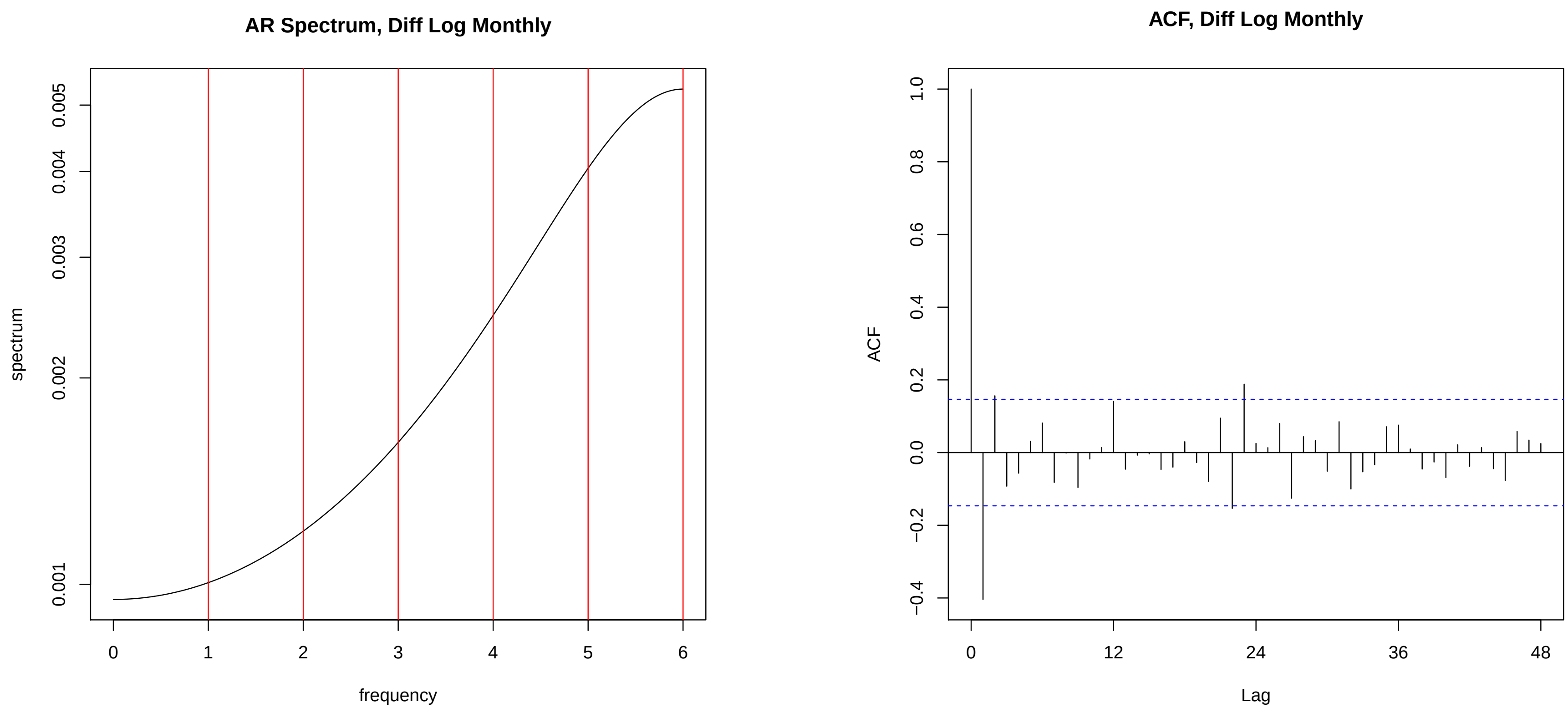


Figure 2: Autoregressive spectrum and autocorrelation function of the differenced log monthly series.

Example 1: Quarterly Aggregate

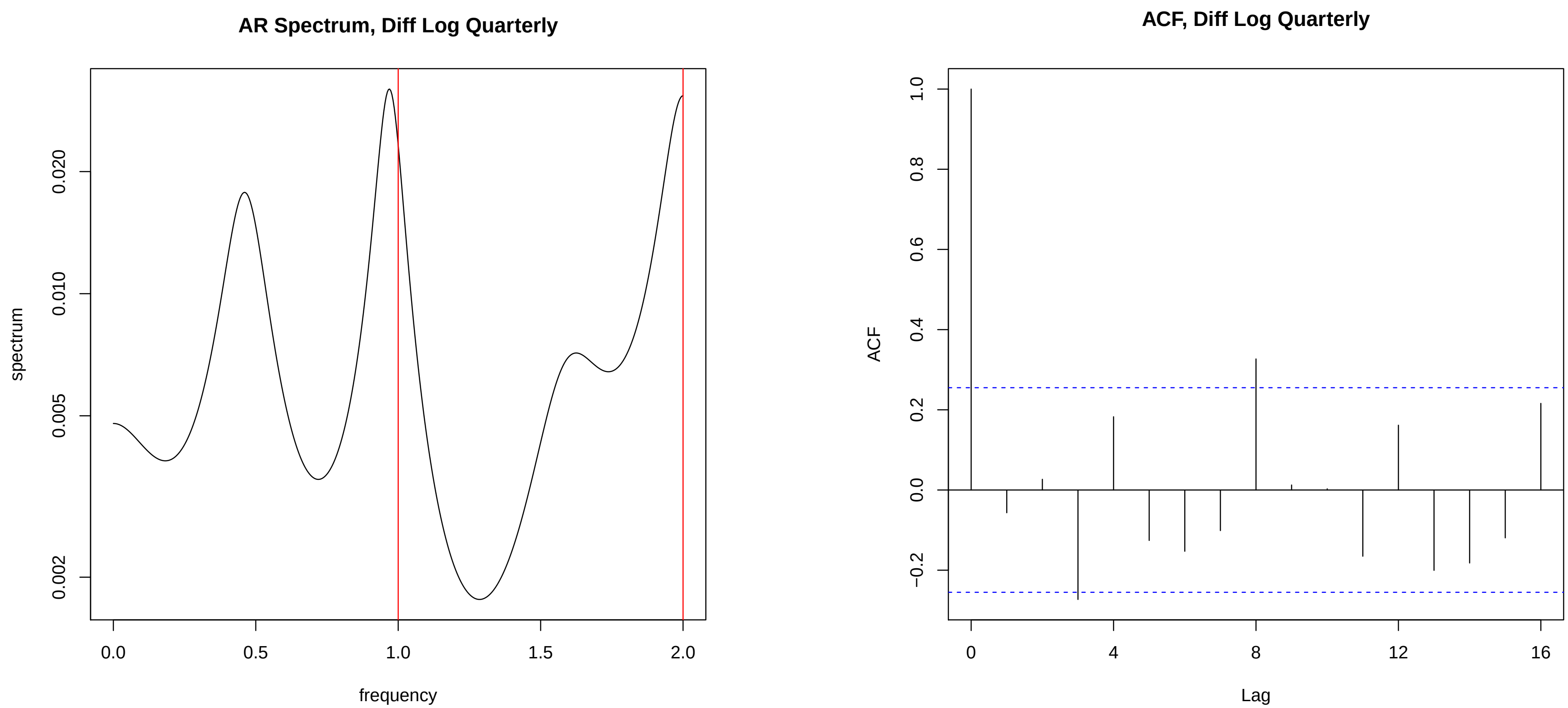


Figure 3: Autoregressive spectrum and autocorrelation function of the differenced log quarterly aggregate.

Example 1: Comments

- AR spectrum for monthly does not show any noticeable peaks; AR spectrum for quarterly has a peak close to quarterly seasonal frequency (red lines)

- ACF for monthly series does not appear to have significant autocorrelations at seasonal lags; ACF for quarterly series appears to show significant autocorrelation at second seasonal lag

- That is, monthly series does not appear to be seasonal (or at least, not noticeably so), but aggregating suggests seasonality may be present at a quarterly frequency

- Table 1 shows values of ρ for which the specified series is deemed seasonal using the root diagnostic (i.e., the series exhibits ρ-persistent seasonality); since monthly series was not adjusted, monthly and monthly SA should be identical, and quarterly aggregate and indirect quarterly seasonal adjustment values should be similar

Series	ρ
Monthly	$\varnothing$
Qtrly Agg	$[0.980, 0.994]$
Monthly SA	$\varnothing$
Indirect Qtrly SA	$[0.980, 0.994]$
Direct Qtrly SA	$\varnothing$
Reconciled Mthly	$\varnothing$
Reconciled Qtrly	$\varnothing$

Table 1: Values of ρ for which the root diagnostic applied to the given series has a p-value exceeding $\alpha = 0.1$.

Example 1: Reconciled Monthly

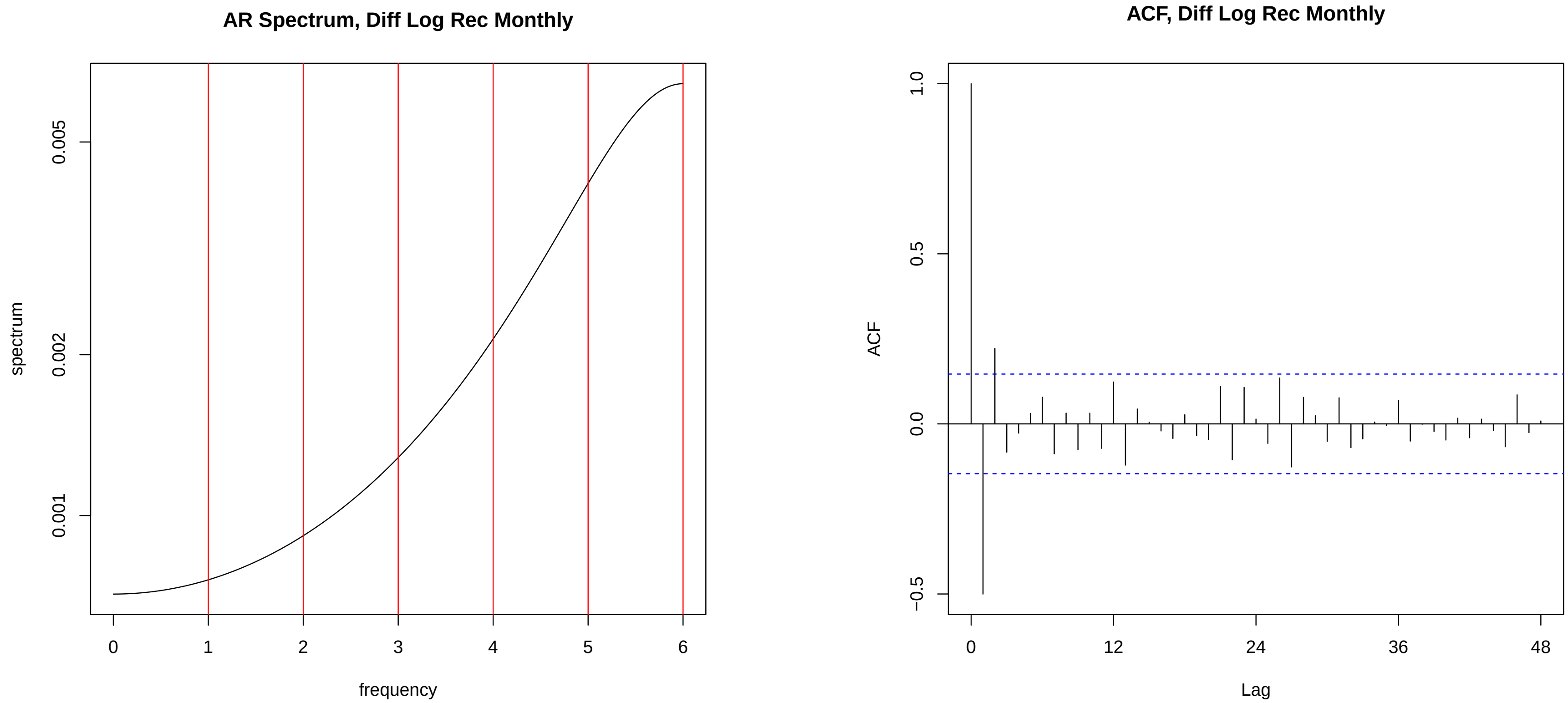


Figure 4: Autoregressive spectrum and autocorrelation function of the differenced log reconciled monthly series.

Example 1: Reconciled Quarterly

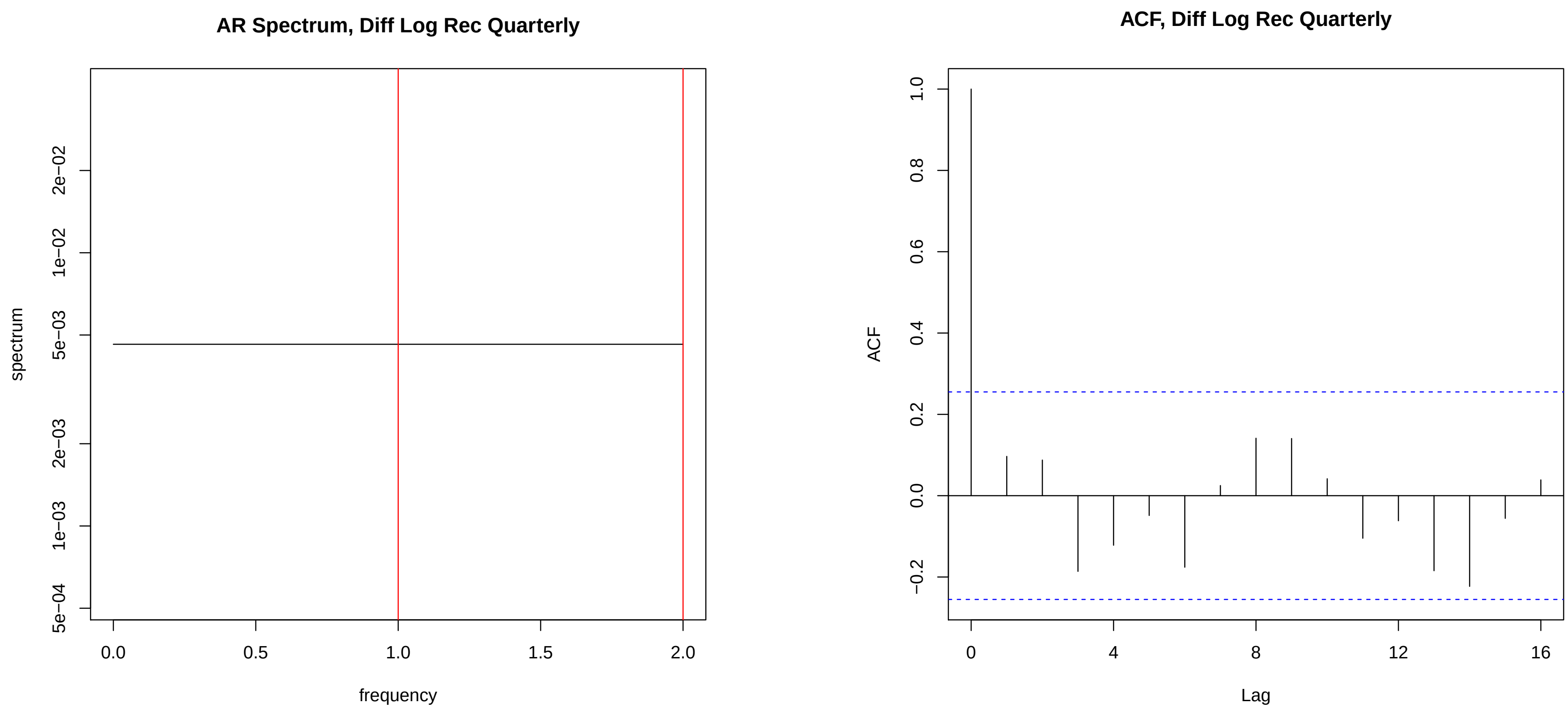


Figure 5: Autoregressive spectrum and autocorrelation function of the differenced log reconciled quarterly series.

Example 1: Monthly and Reconciled Monthly

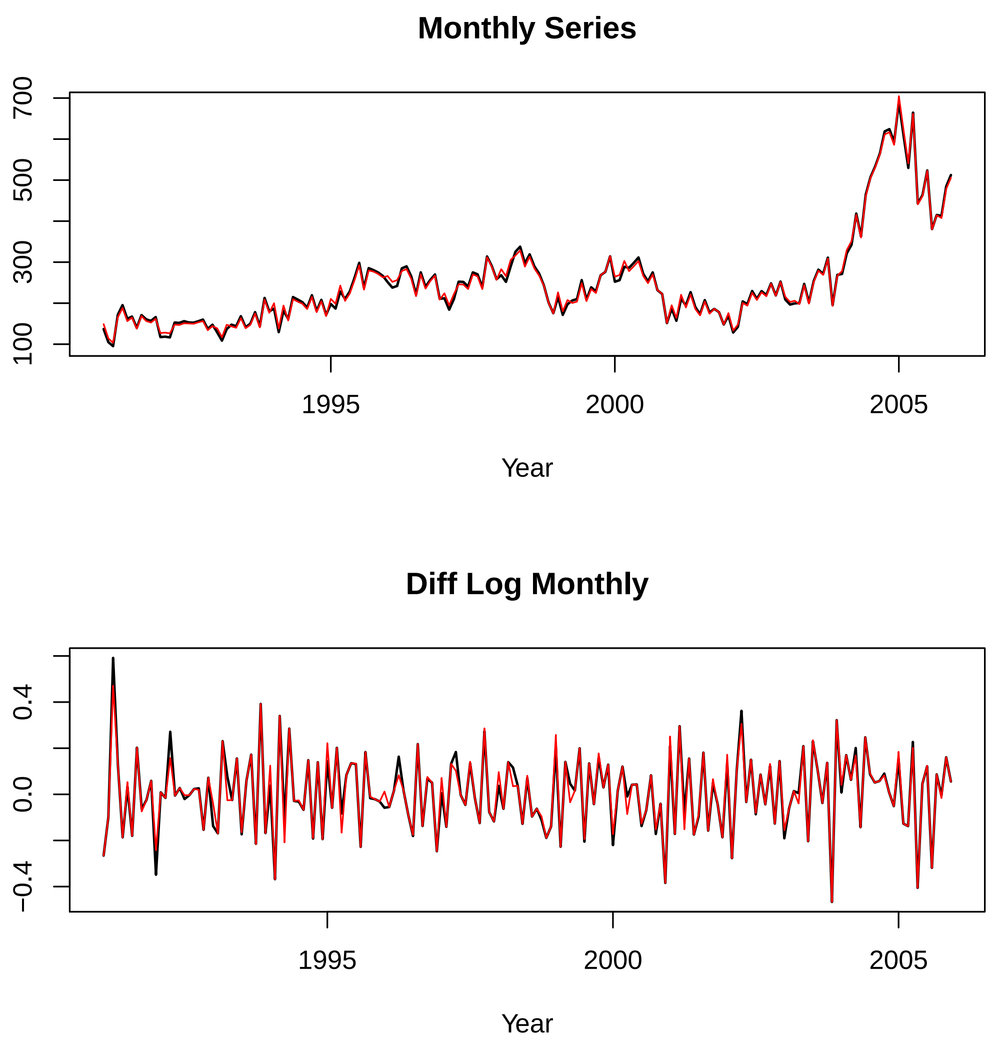


Figure 6: Monthly (black) and reconciled (red) series.

Example 1: Quarterly Aggregate and Reconciled Quarterly

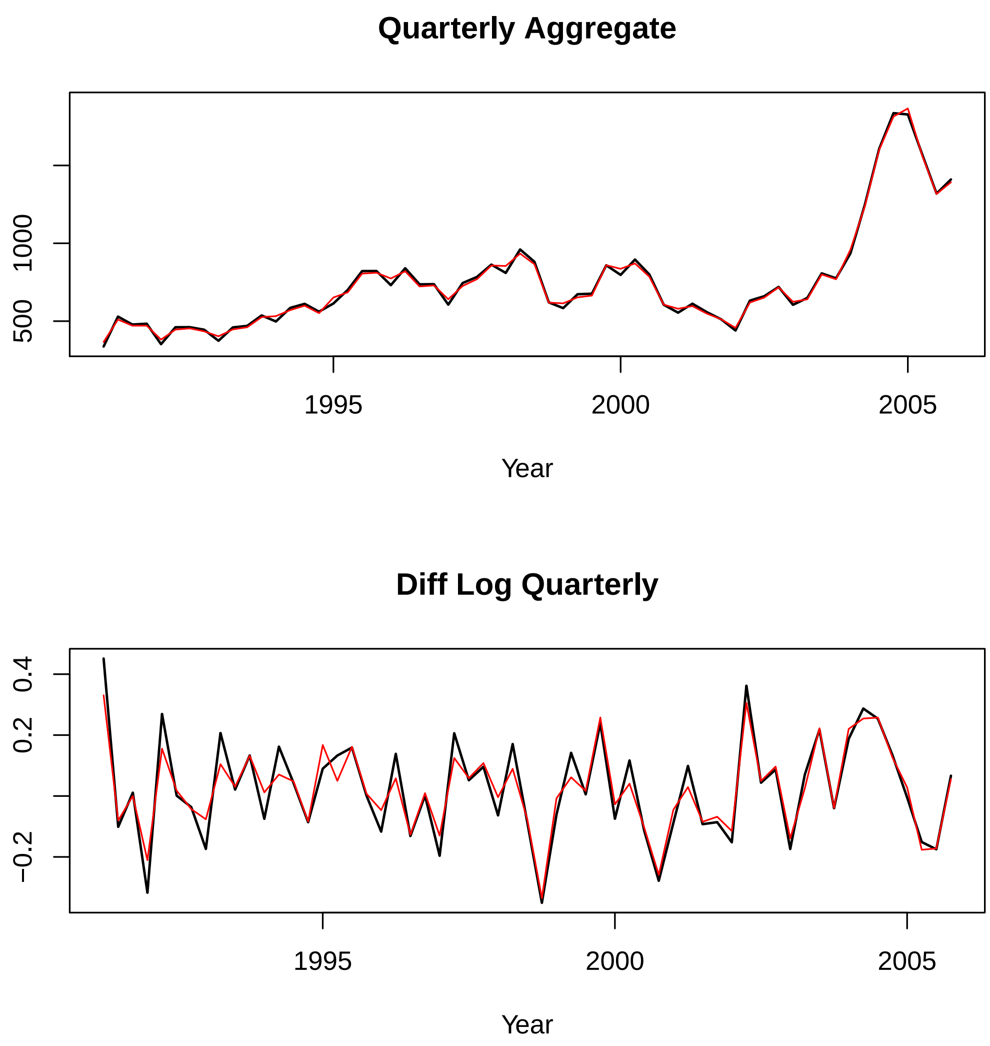


Figure 7: Quarterly aggregated (black) and reconciled (red) series.

United States Census Bureau

Example 1: ... and Direct Quarterly Adjustment

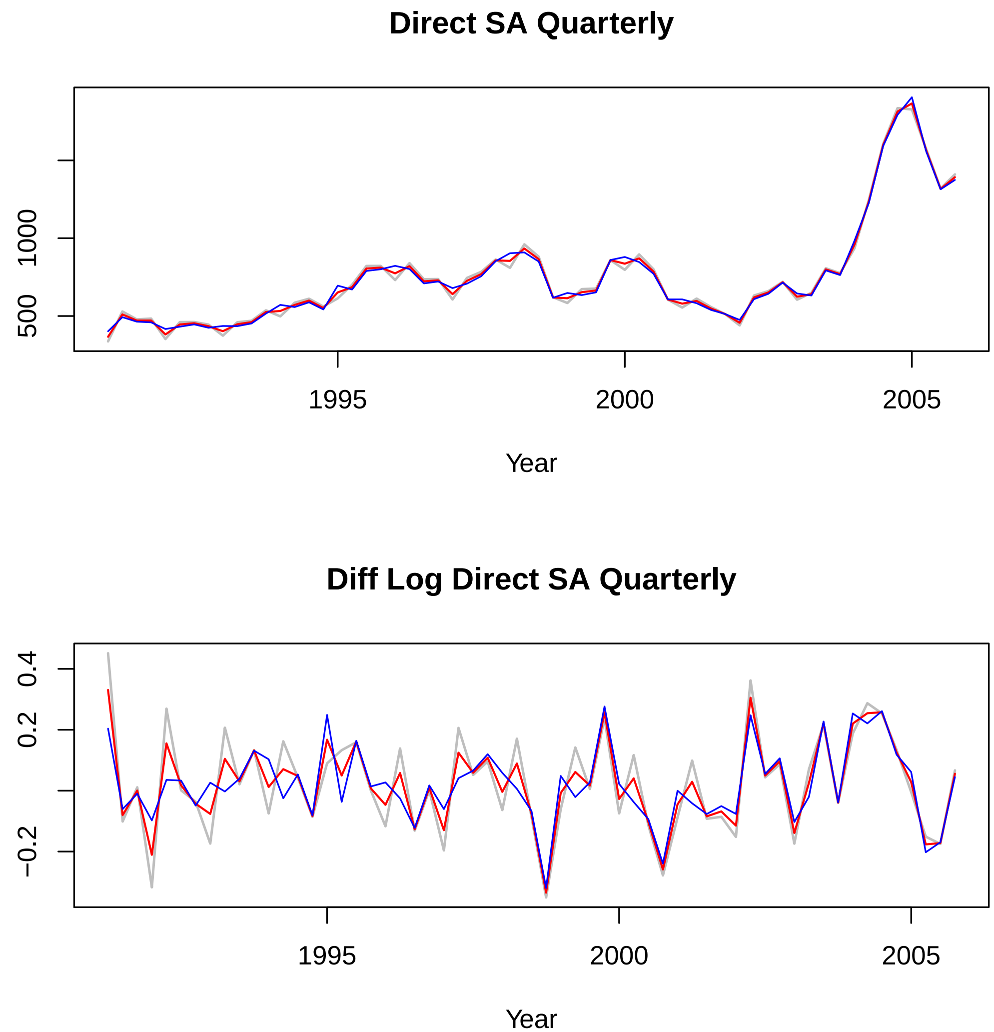


Figure 8: Quarterly aggregated (gray), reconciled (red), and directly adjusted quarterly (blue) series.

Example 2: Construction Spending Series

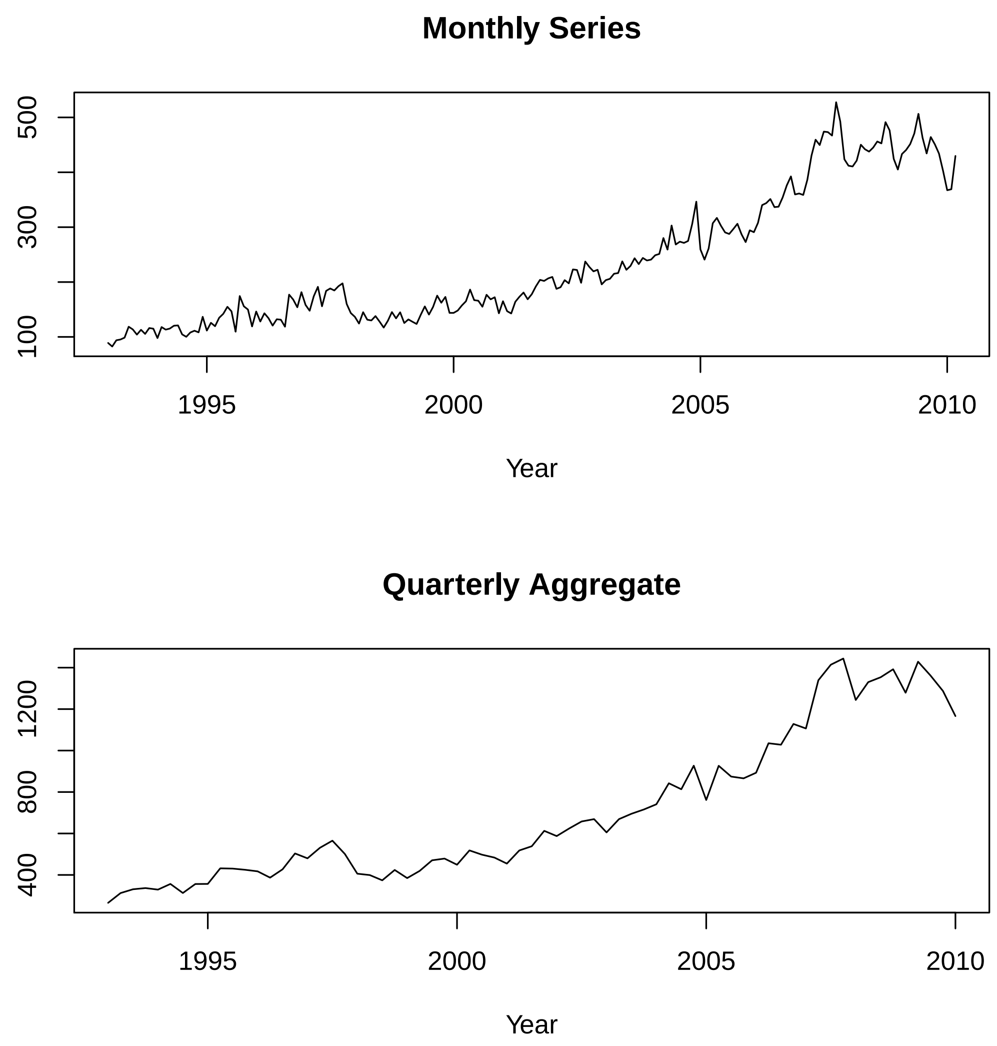


Figure 9: Monthly and quarterly aggregated series.

Example 2: Monthly

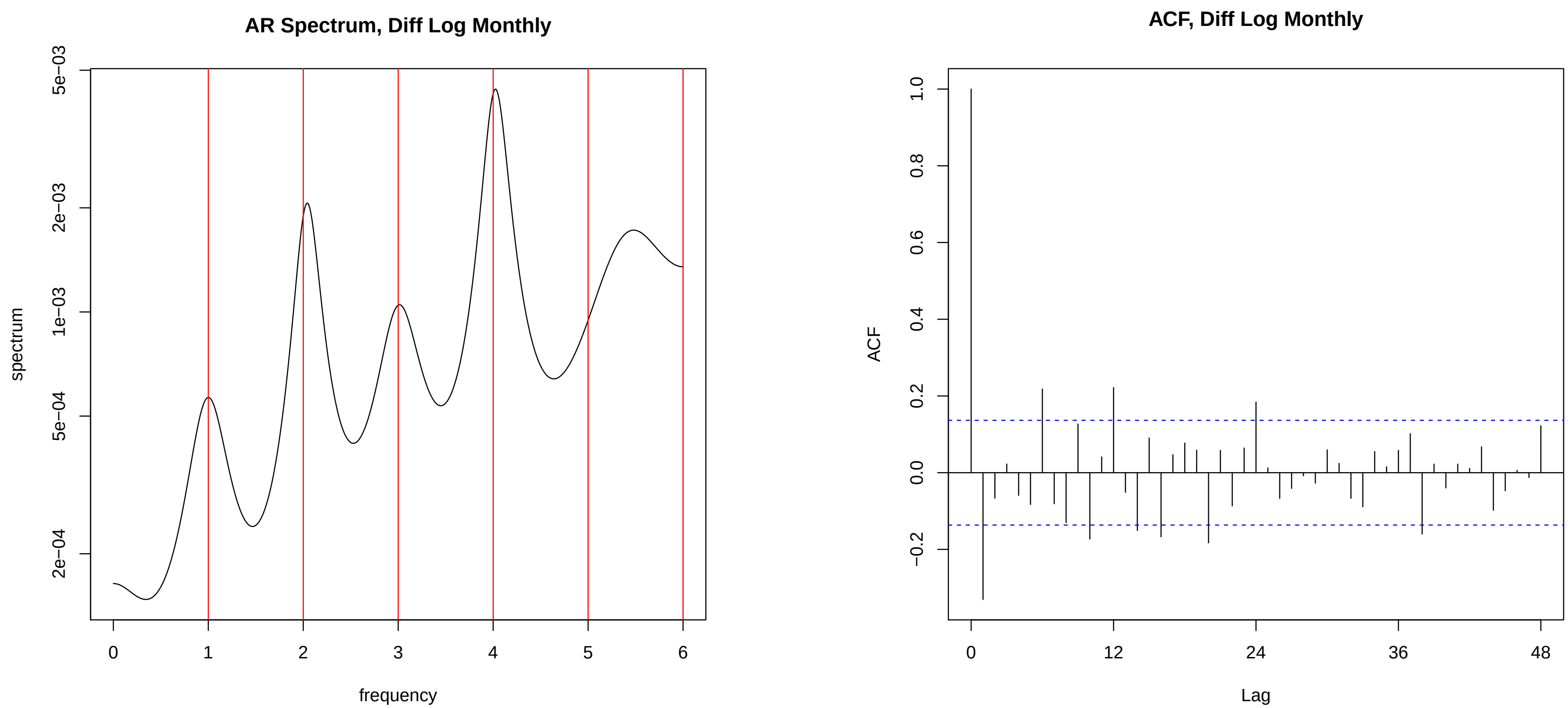


Figure 10: Autoregressive spectrum and autocorrelation function of the differenced log monthly series.

Example 2: Quarterly Aggregate

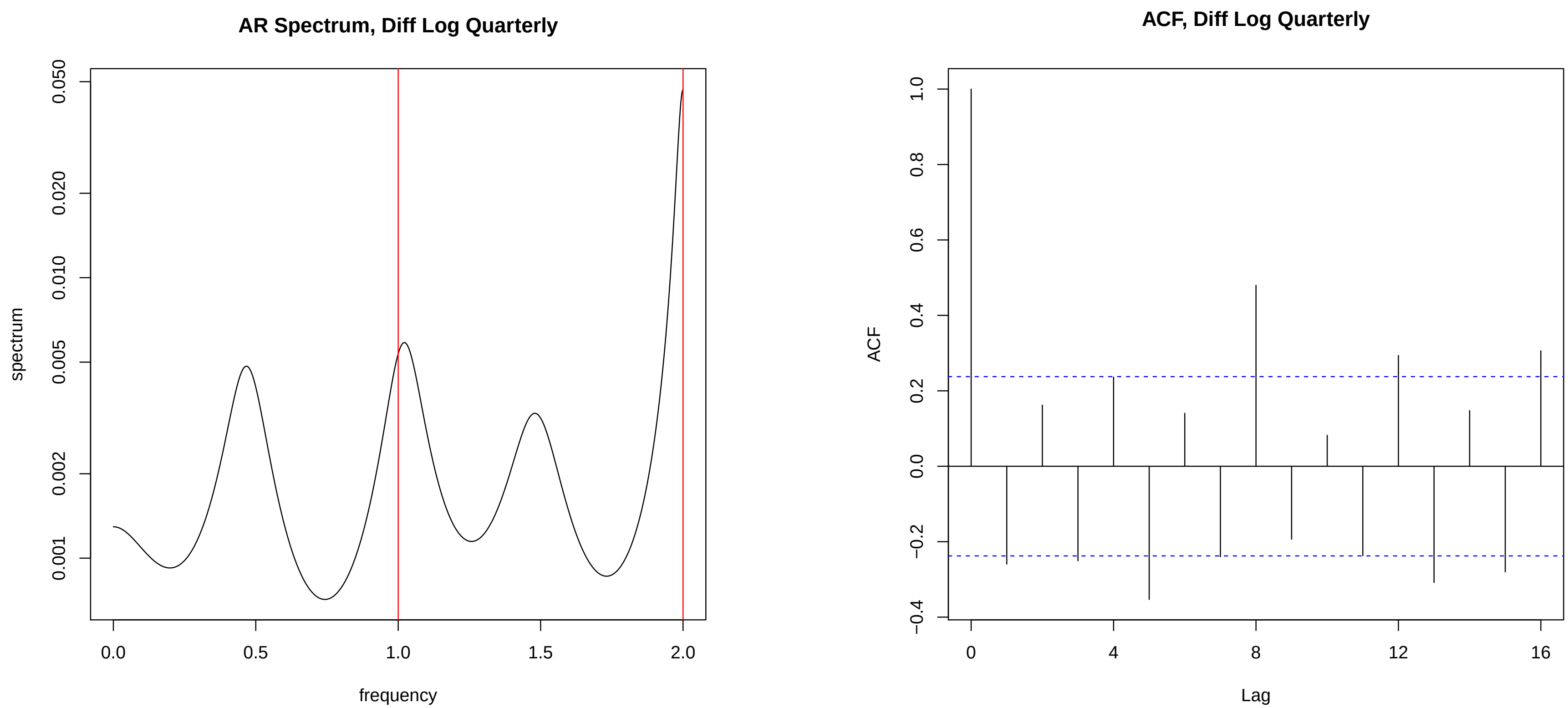


Figure 11: Autoregressive spectrum and autocorrelation function of the differenced log quarterly aggregate.

Example 2: Comments

- AR spectrum for monthly seems like there could be seasonality (i.e, peaks close to 2nd and 4th monthly seasonal frequencies); AR spectrum for quarterly has a peak that appears to be located close to, if not on, quarterly seasonal frequency (red lines)

- ACF for monthly series have autocorrelations that may be significant at 1st and 2nd seasonal lags; ACF for quarterly series shows very large autocorrelation at 2nd seasonal lag, and slightly smaller ones at 3rd and 4th

- That is, seasonality may be present at a monthly level, but seems more noticeable when aggregated to a quarterly frequency

- Table 2 shows values of ρ for which the specified series is deemed seasonal using the root diagnostic (i.e., the series exhibits ρ-persistent seasonality); again, quarterly aggregate and indirect quarterly seasonal adjustment values are similar

Series	ρ
Monthly	$\varnothing$
Qtrly Agg	$[0.980, 0.985]$
Monthly SA	$\varnothing$
Indirect Qtrly SA	$[0.980, 0.985]$
Direct Qtrly SA	$\varnothing$
Reconciled Mthly	$\varnothing$
Reconciled Qtrly	$\varnothing$

Table 2: Values of ρ for which the root diagnostic applied to the given series has a p-value exceeding $\alpha = 0.1$.

Example 2: Reconciled Monthly

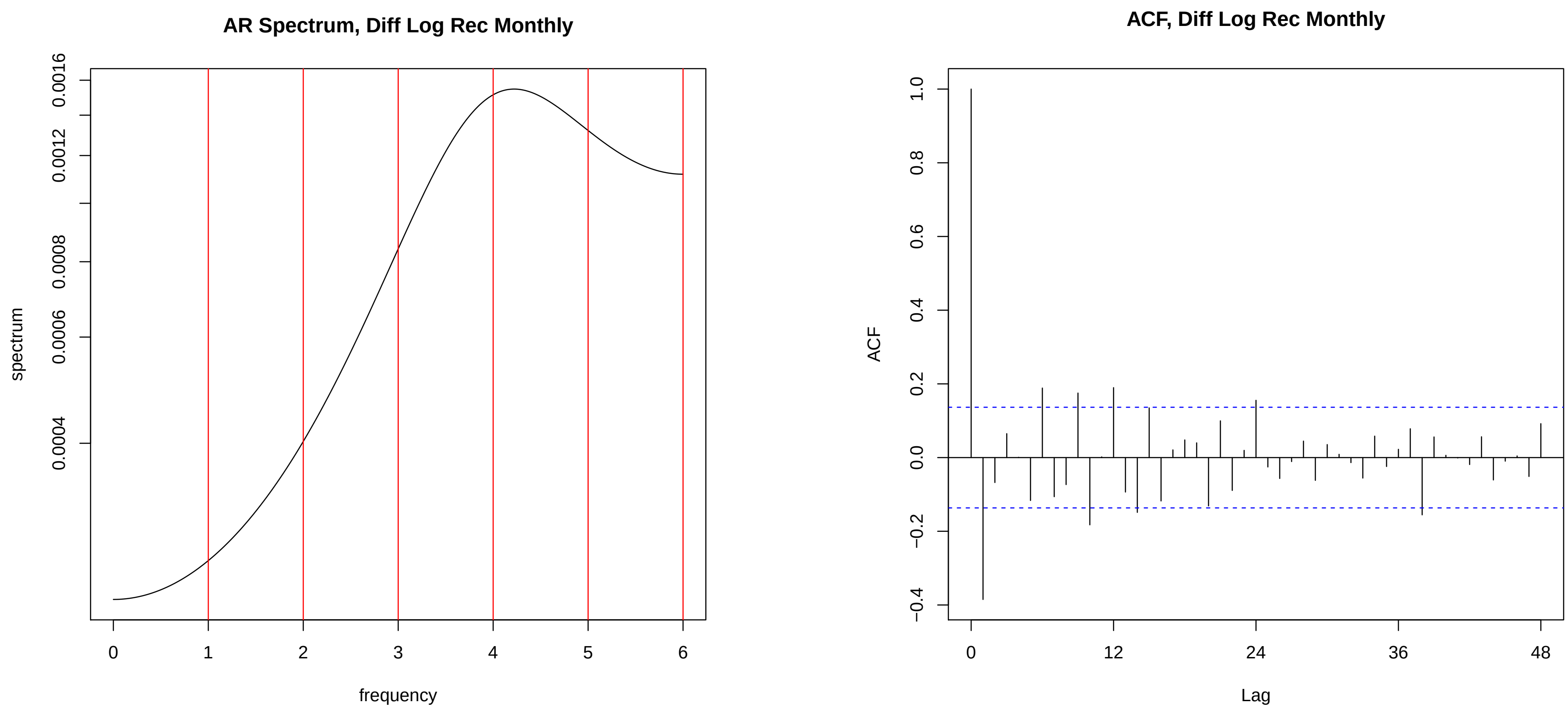


Figure 12: Autoregressive spectrum and autocorrelation function of the differenced log reconciled monthly series.

Example 2: Reconciled Quarterly

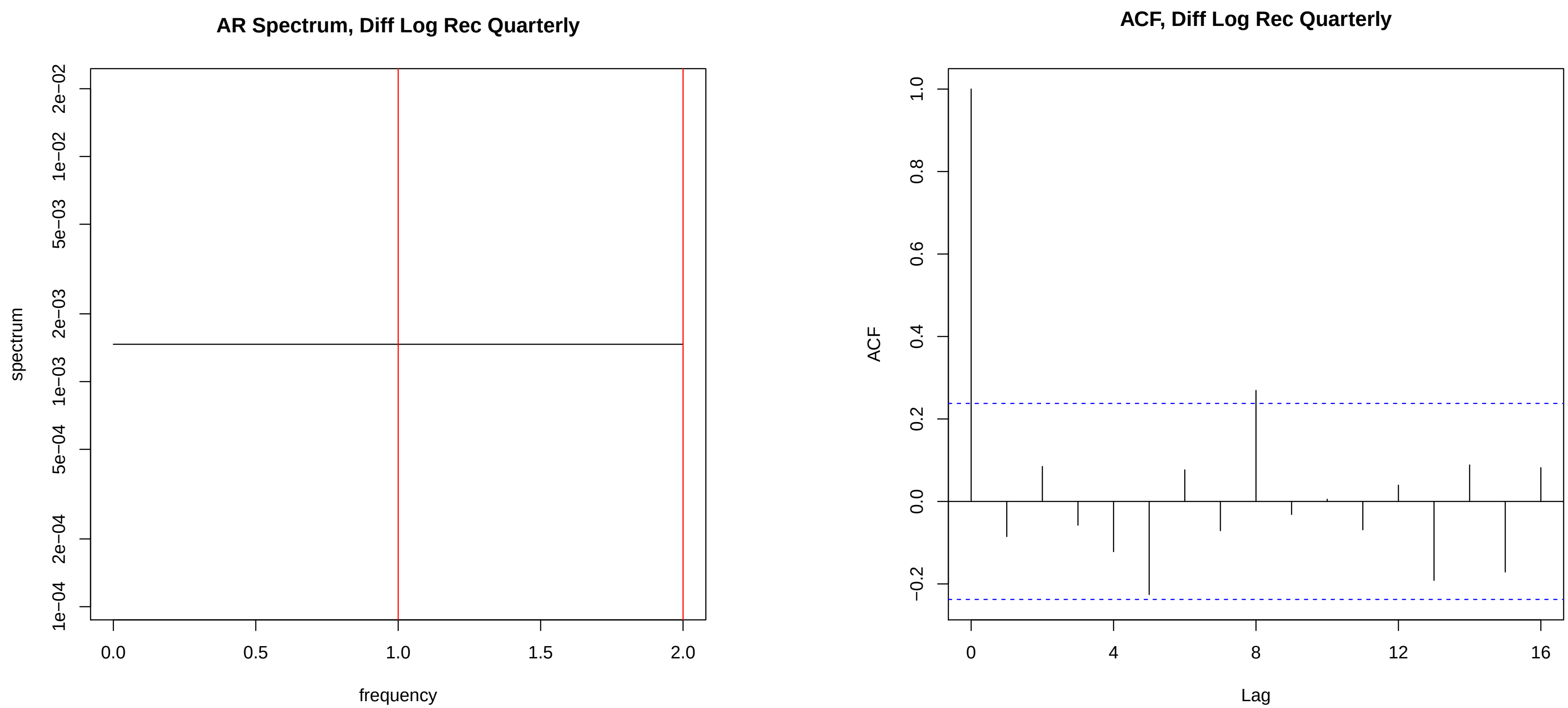
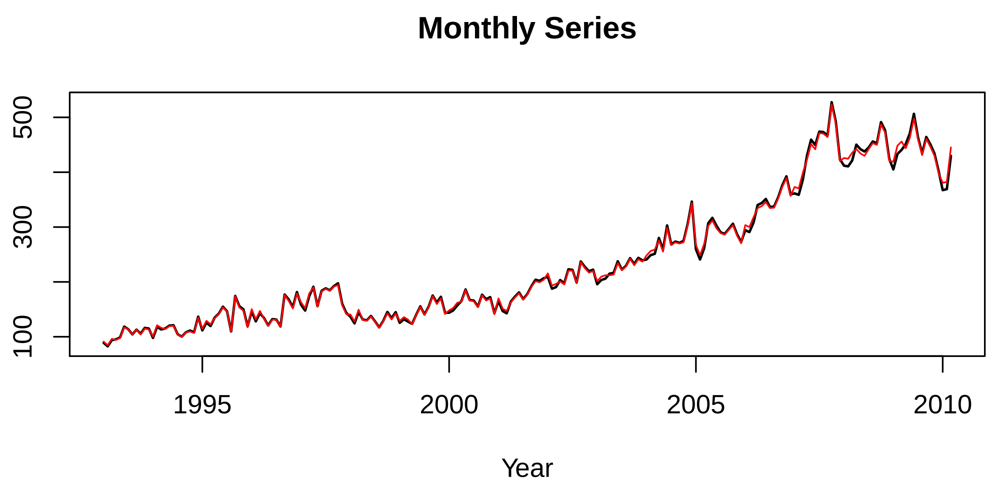


Figure 13: Autoregressive spectrum and autocorrelation function of the differenced log reconciled quarterly series.

Example 2: Monthly and Reconciled Monthly



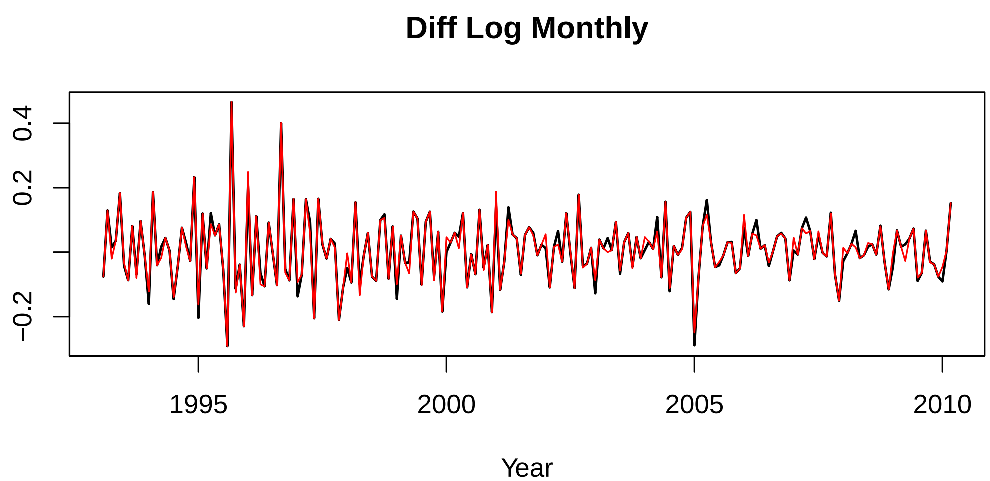


Figure 14: Monthly (black) and reconciled (red) series.

Example 2: Quarterly Aggregate and Reconciled Quarterly

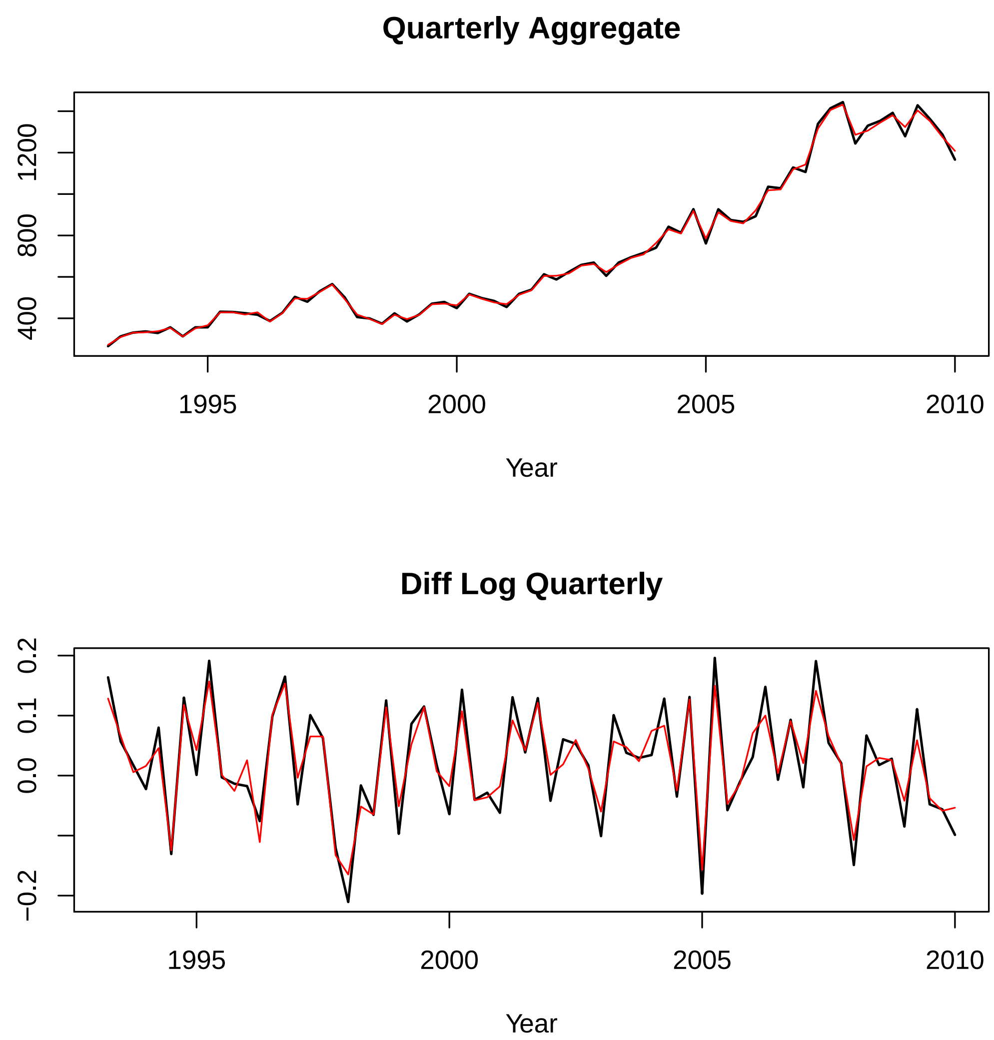


Figure 15: Quarterly aggregated (black) and reconciled (red) series.

Example 2: ... and Direct Quarterly Adjustment

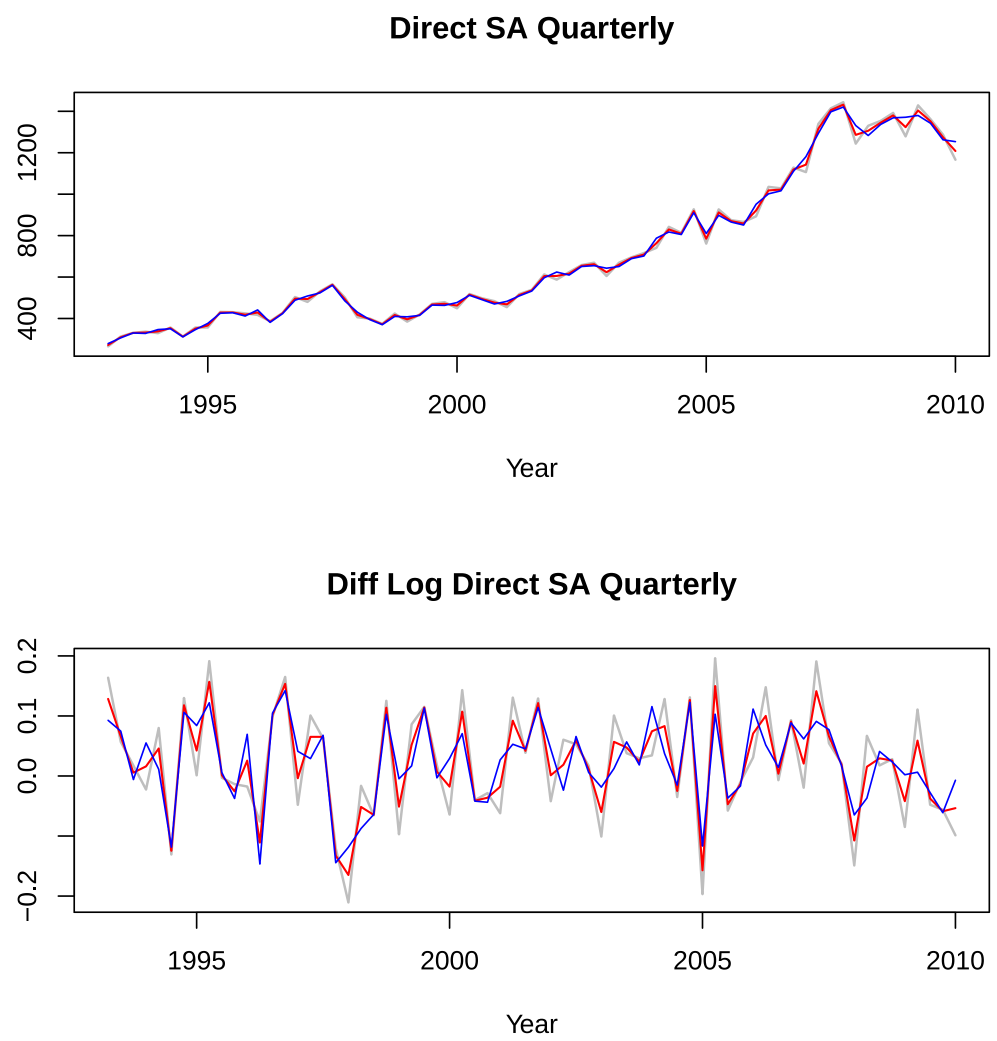


Figure 16: Quarterly aggregated (gray), reconciled (red), and directly adjusted quarterly (blue) series.

An Alternate First Step

- Optimization step is slowest, so one option is to use the exact solution when both ω_m and ω_q have an initial value of 0

- I.e., for any quarter i, with nonseasonal monthly values $N_{3i+1,m}$, $N_{3i+2,m}$, $N_{3i+3,m}$ and quarterly value $N_{i,q}$, the reconciled monthly values $Y_{3i+1,m}$, $Y_{3i+2,m}$, $Y_{3i+3,m}$ can be found by solving

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0.5 \begin{bmatrix} \frac{1}{N_{3i+1,m}} + \frac{1}{N_{i,q}} & \frac{1}{N_{i,q}} & \frac{1}{N_{i,q}} \\ \frac{1}{N_{i,q}} & \frac{1}{N_{3i+2,m}} + \frac{1}{N_{i,q}} & \frac{1}{N_{i,q}} \\ \frac{1}{N_{i,q}} & \frac{1}{N_{i,q}} & \frac{1}{N_{3i+3,m}} + \frac{1}{N_{i,q}} \end{bmatrix} \begin{bmatrix} Y_{3i+1,m} \\ Y_{3i+2,m} \\ Y_{3i+3,m} \end{bmatrix}$$

- This yields a block diagonal structure over the full series, so the reconciled values can be obtained quickly

- If the outcome is not adequate, or if at least one of the initially selected values of ω_m and ω_q is not 0, then the optimization step can be invoked

Thoughts

- Small changes observed to monthly series as a result of this process, but spectra and ACFs look the same (more or less)
- More pronounced changes to quarterly – reconciliation dampens some of the sharper changes in the aggregates
- Proviso: root diagnostic allows for choice of order for ARMA polynomial; examples use order determined by selection criterion (e.g., AIC), but results may vary if fixed values are used instead
- Examples shown here used series where raw monthly was not adjusted and quarterly was more noticeably seasonal
- Expansion to accommodate both frequency and cross aggregation in progress

References

Astolfi, T., D. Ladiray and G.L. Mazzi (2001), Seasonal Adjustment of European Aggregates: Direct versus Indirect Approach, *working paper, Eurostat*, Luxembourg.

Bates, D., K.M. Mullen, J.C. Nash, and R. Varadhan (2014). minqa: Derivative-free optimization algorithms by quadratic approximation. *R package version 1.2.4*. `https://CRAN.R-project.org/package=minqa`

Chen, B. (2012), A balanced system of U.S. industry accounts and distribution of the aggregate statistical discrepancy by industry, *Journal of Business and Economic Statistics*, 30(2), 202–221.

Cholette, P.A. (1984), Adjusting sub-annual series to yearly benchmarks, *Survey Methodology*, 10(1), 35–49.

Dagum, E.B. (1979), On the Seasonal Adjustment of Economic Time Series Aggregates: A Case Study of the Unemployment Rate, Counting the Labor Force, *National Commission on Employment and Unemployment Statistics*, Appendix, 2: 317–344.

Dagum, E.B., and P.A. Cholette (2006), *Benchmarking, Temporal Distribution, and*

Reconciliation Methods for Time Series Data, Springer-Verlag, New York. Lecture Notes in Statistics 186.

Denton, F. (1971), Adjustment of monthly or quarterly series to annual totals: an approach based on quadratic minimization, *Journal of the American Statistical Association*, 82, 99–102.

Di Fonzo, T. and M. Marini (2011), Simultaneous and two-step reconciliation of systems of time series: methodological and practical issues, *Journal of the Royal Statistical Society C*, 60(2), 143–164.

Findley, D.F., B.C. Monsell, W.R. Bell, M.C. Otto, and B.C. Chen (1998), New capabilities and methods of the X-12-ARIMA seasonal-adjustment program, *Journal of Business and Economics Statistics*, 16, 127–177.

Findley, D.F., D. Lytras and T.S. McElroy (2017), Detecting seasonality in seasonally adjusted monthly times series, *Census Bureau Research report* 2017-03.

Franses, P.H. (1994) A multivariate approach to modeling univariate seasonal time series. *Journal of Econometrics* **63**, 133–151.

Furman, J. (2015), Second Estimate of GDP for the First Quarter of 2015. *Council of Economic Advisers Blog*, May 29, 2015.

Gilbert, C.E., N.J. Morin, A.D. Paciorek, and C.R. Sahm (2015), Residual Seasonality in GDP, *FEDS Notes*, May 14, 2015.

Gladyshev, E.G. (1961) Periodically correlated random sequences. *Soviet Mathematics* **2**, 385–388.

Groen, J. and P. Russo (2015), The Myth of First-Quarter Residual Seasonality, *Liberty Street Economics*, June 8, 2015.

Hood, C.H. and D.F. Findley (2001), Comparing Direct and Indirect Seasonal Adjustments of Aggregate Series, *American Statistical Association Proceedings of the Business and Economics Statistics Section*.

Kuhn, H. W. and Tucker, A. W. (1951) Nonlinear programming. *Proceedings of 2nd Berkeley Symposium*. Berkeley: University of California Press, 481–492.

Maravall, A. (2012), Update of Seasonality Tests and Automatic Model IdentiÂĚcation in TRAMO-SEATS, *Bank of Spain (November 2012 draft)*.

McElroy, T. (2016) Challenges with seasonal adjustment, *FESAC meeting slides*. `https://www.census.gov/about/adrm/fesac/meetings/2016-12-09-meeting.html`

McElroy, T. (2018) Seasonal adjustment subject to accounting constraints. *Statistica Neerlandica* **72**, 574–589.

McElroy, T. (2019) A diagnostic for seasonality based upon autoregressive roots. *Mimeo*.

Moulton, B. (2016), Residual seasonality in GDP and GDI, *Federal Economic Statistics Advisory Committee*, December 9, 2016.

Osborn, D.R. (1991) The implications of periodically varying coefficients for seasonal time-series processes. *Journal of Econometrics* **28**, 373–384.

Powell, M.J. (2009), The BOBYQA algorithm for bound constrained optimization without derivatives, *Report No. DAMTP 2009/NA06*, Centre for Mathematical Sciences, University of Cambridge, UK. http://www.damtp.cam.ac.uk/user/na/NA_papers/NA2009_06.pdf.

Quenneville, B. and S. Fortier (2012), Restoring Accounting Constraints in Time Series ÂŰMethods and Software for a Statistical Agency, in *Economic Time Series: Modelling and Seasonality*, edited by W. R. Bell, S. H. Holan and T. S. McElroy, Boca Raton: CRC Press, Taylor and Francis Group.

Rudebusch, G.D., D. Wilson and T. Mahedy (2015), The Puzzle of Weak First-Quarter GDP Growth, *FRBSF Economic Letter*, May 18, 2015.

United States Census Bureau

Smith, A.E. and Coit, D.W. (1995) Penalty functions, in *Handbook of Evolutionary Computation*, 97(1), p.C5.

Tiao, G.C. and Grupe, M.R. (1980) Hidden periodic autoregressive-moving average models in time series data. *Biometrika* **67**, 365–373.